DOCUMENT RESUME

ED 197 287 CS 005 810

AUTHOR Sullivan, Kathryn
TITLE The Comprehension of Mathematical Language, a Communication Approach.
PUB DATE Dec 80
NOTE 15p.: Paper presented at the Annual Meeting of the American Reading Conference (1st. Sarasota, FL, December 4-6, 1980).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Content Area Reading: Elementary Secondary Education: *Mathematical Formulas: *Mathematical Vocabulary: Mathematics Instruction: *Reading Comprehension: *Reading Processes: *Reading Skills

ABSTRACT

One framework for reading instruction that focuses on the message suggests that all words can be classified as matter or relation references. This framework is applicable to the words and symbols used in mathematical symbolic sentences when they are defined as two or more names for numbers of sets joined together by a relation symbol. The use of this matter-relation class framework to teach students to determine the message intended by math symbolic sentences can make math a meaningful communication rather than a series of symbols committed to memory. However, while using the concepts of matter and relation words and helping children perceive the mathematics symbolic sentences as a form of communication in which the sender is locating matter references using relation references, care must be taken to insure that the approach does not become a matter of memorization. (HOD)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Kathryn Sullivan
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

The Comprehension of Mathematical Language, A Communication Approach

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

Presented by Kathryn Sullivan, Pembroke State University
Presented at the American Reading Conference, December 5, 1980

ED197287

INTRODUCTION

If newspapers, magazines, and professional journals may be considered representative of the educational concerns of our country then, reading, writing, and arithmetic test scores are certainly in a priority position. Whether scores are actually declining, remaining the same, or increasing slightly, education has been hit with a "back-to-basics hysteria" (Greider, 1979, p. 24) and schools are being asked to focus once again on the teaching of reading, writing, and arithmetic*. In most classrooms reading and mathematics are considered separate subjects with little carry-over from one to the other except in the area of word problems. The purpose of this paper is to suggest a means of relating these two areas and ultimately enhancing the students' comprehension and understanding of mathematics.

RATIONALE

Although opinions as to the exact definition of reading may differ, most educators would agree that the reading process involves the perception, interpretation and evaluation of the printed symbol (Lapp & Flood, 1978). Carroll (1976) suggested that "the essential skill in reading is getting meaning from a printed or written message." (p. 11). With today's emphasis on making mathematics more than a matter of rote memorization where "2 'n 2 'r 4" and "4 'n 4 'r 8" mathematics in essence may become an aspect of reading as it

*throughout this paper the term mathematics will be used so as to include not only the basic operations but geometry, algebra, set theory, etc.

CS205870

involves the perception, the interpretation, and the evaluation of a printed symbol. When one reads the symbol "cat" hopefully an image of the appropriate animal is evoked. Likewise, when one reads the symbol "2" hopefully an appropriate image of more than one but less than three is evoked. Are the two processes as distinct and discrete as we tend to treat them in the separation of reading and mathematics? When children are able to ascertain the meaning of symbols such as "Dogs and cats are animals." or "Boys and girls are children." we say they are reading. When they are able to ascertain the meaning of the symbols "2 + 4 = 6" or "7 - 3 = 4" we say they are doing math. Is there really a difference?

In reading, the cloze procedure may be used to evaluate or teach comprehension. Sometimes there is only one correct answer as in the case of "__ am in the room." and sometimes there are a variety of correct answers as in the case of "The ____ chased the cat." Similarly, mathematics uses expressions such as "2 + ___ =7" which have basically one correct answer although it may be indicated by different symbols (e.g. 5, 10/2, 20/4) and expressions such as "2 + __ > 7" which have a variety of answers. Again, one might ask if there really is a difference.

Examples such as the preceding would indicate that the perception, interpretation, and evaluation of mathematical symbols is not a distinct and separate process. Why then is the reading of symbols such as "2" and "2 + 2=4" treated differently from the reading of the symbols "dogs" and "Dogs and cats are animals."?

It has been suggested that to many people mathematics is essentially "something done in school according to someone else's rules" (Jencks & Peck, 1975, p. 371). Math makes little sense and becomes very similar to word calling. One can accurately pronounce all the symbols but has little understanding of the underlying message, little comprehension.

Hands-on experiences have been advocated but something is still missing. What appears to be needed is a method which focuses on the message of such sentences as "2 + 3 = 5" and "6 - 4 = 2". A method which focuses on the communication symbolized by the sentence.

COMMUNICATION APPROACH

In recent years Dinnan and Lodge (1976) have outlined a framework for reading instruction which focuses on the message, the communication represented by the printed message. They suggest that all words can be classified as matter or relation references. Matter references being those referring to objects which have volume and weight such as "boy", "chair", and "lamp", while relation references indicate time, space, amount, and quality. Sample time references include "is", "was", and "then"; sample space references include "in", "out", and "near"; sample amount references include "one", "some", and "all"; sample quality references include "good", "bad", and "well". (This classification is depicted in Figure 1.) Dinnan and Lodge further suggest that communication basically involves the location of matter references in terms of time, space, amount, and quality relations. The more definitely the sender (be he speaker or writer) locates the matter references for the receiver, the more accurate the communication. Those references not defined by the sender must be supplied by the receiver and can result in miscommunication.

Students are taught to identify and manipulate matter and relation references in order to ascertain the intended message. Using this approach, reading is a meaningful process of identifying relationships, a communication process.

THE READING OF MATHEMATICS

In order to determine whether or not this framework could be applied

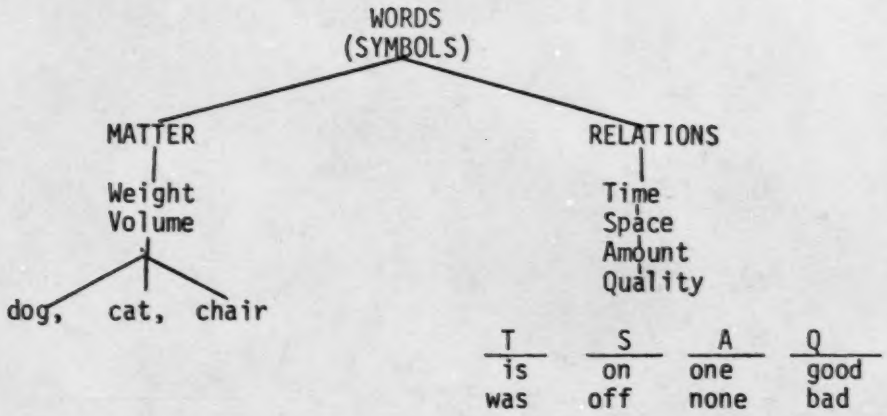

Figure 1. Graphic representation of Dinnan and Lodge (1976) matter-relation references.

to mathematical language and subsequently symbolic sentences an investigation of the language (both words and symbols) of seven mathematics series published within the last five years was conducted (Sullivan, 1980). Using the definition of mathematical symbolic sentences offered by Heddens (1971) which suggested that symbolic sentences are expressions such as "2 + 3 = 5" consisting of "two or more names for numbers or sets joined together by a relation symbol" (p.138) the investigator proceeded through forty-nine textbooks recording the various symbolic sentences found there and the words used to "orally" read them. (Table 1 contains several examples.) These sentences were then used to ascertain the relations (same-different and many- few) between the words and symbols used in mathematical symbolic sentences and the Dinnan and Lodge (1976) matter-relation classes.

The analysis indicated that the matter and relation classes defined by Dinnan and Lodge (1976) were applicable to the words and symbols used in mathematical symbolic sentences. (See Tables 2 and 3 for representative examples.) The number symbols and the words used to call them be they known values such as "2" or unknown values such as "y" represent the matter class. The remaining symbols represent relations. It should be noted that the matter references used in mathematical language represent matter in general and are not tied to specific objects. For example, in the sentence "1 + 2 = 3", "2" can be horses, cows, rocks, people, buildings, or whatever matter one might care to specify. The relation references needed in mathematics are fewer in number than the relation references needed in ordinary language. While one might need the relations of "sick-well" and "beautiful-ugly" in ordinary language these are not essential to mathematical language. This is depicted in Figure 2.

The next apparent question was whether or not these matter and relation references are used similarly in ordinary language and mathematical language.

Table 1

Sample Recordings of Symbolic Sentences

and the Words Used to Orally Read Them

Symbolic Sentence	Oral Reading
1 + 2 = 3	one plus two equals three
	one add two equals three
	one plus two are three
	one and two is the same as three
5 - 1 = 4	five minus one equals four
	five take one is four
	five take-away one is four
	five minus one is the same as four

Table 2

Classification of the Words Suggested in the Textbooks to be Used in Reading Selected Symbolic Sentences

Symbolic Sentence	Words Suggested for Oral Reading of the Sentences and their Classifications
1 + 2 = 3	M* r M* r M* One plus two equals three. M* r M* r M* One and two are three.
7 > 3	M* r M* Seven is greater than three.
3 x 4 = 12	M* r M* r M* Three times four is twelve. M* M* r M* Three fours equal twelve.
∠ABC ≅ ∠DEF	M* r M* Angle ABC is congruent to Angle DEF
B ⊆ A	M* r M* Set B is a subset of Set A
4 qts. ≈ __L	M* r Four quarts is approximately equal to M* how many liters?

M* represents mathematical matter reference, r represents relation reference.

Table 3

Representative Symbolic Sentences and Symbol Classifications

M* r M* r M* 1 + 2 = 3	M* r M* r $\frac{13}{4}$ = $3\frac{1}{4}$ r
M* r M* r M* 5 - 1 = 4	M* M* M* r M* ∠CAB = ∠DEF
M* r M* r M* 7 □ 1 = 8	M* r M* R → M
M* r M* 7 > 3	M* r M* Б ⊆ A
M* r M* r M* 3 x 4 = 12	M* r M* 10 dm = ____cm
M* r M* r M* r M* (3 x 4) + 3 = □	M* r M* r M* X + 4 > 9
M* r M* r M* 6 ÷ 2 = 3	M* r M* r M* .8 = $\frac{2y + 6}{10}$ r M*

M* represents mathematical matter reference, r represents relation reference.

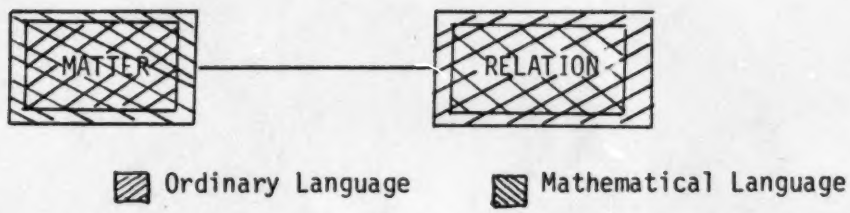

Figure 2. Graphic representation of the relationship between the matter/relation references used in ordinary language sentences and mathematical symbolic sentences.

Dinnan and Lodge (1976) suggest that sentences are expressions of relations and that within a sentence relation references are used to locate matter. Analysis of a representative sample of mathematical symbolic sentences suggested that the same holds true for mathematics. Symbolic sentences contain time, space, amount, and quality references, and these relation references appear to be used to locate the matter references. Table 4 contains sample sentences from arithmetic, geometry, and algebra in which the matter and relation references have been identified.

IMPLICATIONS

It has been noted that to many people mathematics is "something done in school according to someone else's rules" (Jencks & Peck, 1975, p. 371). Use of the matter-relation class framework to teach students to determine the message intended by math symbolic sentences appears to have the potential to make math a meaningful communication rather than a series of symbols committed to memory. However, while using the concepts of matter and relation words and helping children perceive the mathematics symbolic sentences as a form of communication in which the sender is locating matter references (in this case "1", "2", and "3", etc.) using relation references (in this case "=", "+", "⊥", and ">", etc.) care must be taken to insure that the approach does not become a matter of memorization. With this in mind, the following sequence for using this approach is suggested:

1. Establish the concepts of matter and relation references and their functioning in ordinary language.
2. Establish the specific contrasts needed to manipulate the mathematical symbolic sentences. Table 5 contains a listing of 50 words which were found to represent 51.5% of the 44,000 sampled by Sullivan (1980). These words might be considered a starting point. Caution is needed with those words marked with an asterisk. Discrepancies between the ordinary language and mathematical language meanings of those words

Table 4

Location of Time, Space, Amount, and Quality Relations

in Mathematical Symbolic Sentences

Sentences	References
$3 + 2 = 5$ $5 - 3 = 2$ $3 \times 4 = 12$ $6 \div 2 = 3$	Matter: Matter is represented by the symbols "2", "3", "5", etc. Time: The order and sequence relations between the matter references are signaled by the spatial position of the symbols to each other. Space: A together-apart relation is indicated by the "+", "-", "x", and "÷" signs. These signs signal the direction of matter references relative to each other. Amount: The amount relations in these sentences are represented by the "=" sign and reflected in the number of matter references being located. Quality: A quality relation is not always present in these sentences. When it is present, it is reflected in the ">" and "<" signs.
$.8 = \frac{2y + 16}{10}$ $3X + 3 = 24$	Matter: Matter is represented by such symbols as ".8", "2y", and "3X". Time: As in the previous sentences, time is reflected by the spatial relations of the symbols to each other. Space: A together-apart relation is indicated by the signs "+", "x", "-", and "—". These symbols signal the direction of the movement of the matter references relative to each other. Amount: Amount references are reflected in the equality (=) sign and the number of matter references present in the sentences.

Table 4 (Continued)

Sentences	References
	Quality: Although not present in all sentences, quality references are represented by such symbols as ">" and "<".
$\Delta CAB \cong \Delta DEF$	Matter: Matter is represented by such symbols as CAB, DEF, and AB.
$\overline{AB} \perp \overline{CD}$	Time: No explicit reference to time is contained in these sentences. However, the words used to read such signs as "$\cong$" and "$\perp$" suggest it is the present time.
	Space: The spatial relations between the matter being considered is reflected in signs such as "$\perp$" or "$//$". The "Δ" and "$\overline{\ }$" symbols reflect the spatial relations within the matter references.
	Amount: The amount relations are signaled by the "=" sign and the number of matter references present in the sentence.
	Quality: The quality relation within the matter reference itself is implicit in symbols such as "$\angle$", "Δ", and "$\leftrightarrow$".

Table 5

Rank Order Listing of Top 50 Words

the	can	that
*of	this	find
*to	tens	ten
*is	lock	set
a	each	five
and	*on	as
*one	three	us
*are	it	or
*in	we	*with
*how	ones	there
*many	page	numbers
what	your	use
two	*by	numeral
you	write	which
*at	hundred	*more
*for	four	do
number	who	

were found to exist.

3. Teach the students to identify the matter and relation references in mathematical symbolic sentences.

4. Teach the students to interpret and manipulate the references they have identified.

REFERENCES

Carroll, J.B. The nature of the reading process. In H. Singer & R. Ruddell (Eds.), Theoretical models and processes of reading (2nd ed.). Newark, Del.: International Reading Association, 1976.

Dinnan, J.A. & Lodge, R. Communication: A meta theory of language. Athens, Ga.: Jaddy Enterprises, 1976.

Greider, W. Stop knocking our public schools. Today's Education, 1979, 68, 23-24.

Hedden[illegible] J.W. Today's mathematics, a guide to concepts and methods in e[illegible]mentary school mathematics (2nd ed.). Palo Alto, Cal.: SRA, 1971.

Jencks, S., & Peck, D. Symbolism and the world of objects. Arithmetic Teacher, 1975, 22, 370-371.

Lapp, D., & Flood, J. Teaching reading to every child. New York: Macmillan Co., 1978.

Sullivan, K.M. An analysis of the mathematical language of selected K-6 grade mathematics texts utilizing a communication theory of language. Unpublished doctoral dissertation. University of Georgia, 1980.